Trapping Abelian anyons in fractional quantum Hall droplets

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We study the trapping of Abelian anyons (quasiholes and quasiparticles) by a local potential (e.g., induced by an atomic force microscopy tip) in a microscopic model of fractional quantum Hall liquids with long-range Coulomb interaction and edge confining potential. We find, in particular, that at Laughlin filling fraction $\nu=1/3$, both quasihole and quasiparticle states can emerge as the ground state of the system in the presence of the trapping potential. As expected, we find that the presence of an Abelian quasihole has no effect on the edge spectrum of the quantum liquid, unlike in the non-Abelian case [X. Wan *et al.*, Phys. Rev. Lett. **97**, 256804 (2006)]. Although quasiholes and quasiparticles can emerge generically in the system, their stability depends on the strength of the confining potential and the strength and the range of the trapping potential. We discuss the relevance of the calculation to the high-accuracy generation and control of individual anyons in potential experiments, in particular, in the context of topological quantum computing.

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I. INTRODUCTION

Shortly after the discovery of the fractional quantum Hall effect,[1] Laughlin[2] realized that electrons in such a system form an incompressible quantum liquid with excitations of fractional charge.[3,4] These exotic particle excitations[5,6] are dubbed (Abelian) anyons. Interchanging two anyons, one obtains a phase factor $e^{i\theta}$ for the wave function, where θ is neither an integral multiple of 2π as required by bosons nor an odd multiple of π as required by fermions. The presence of these particles with fractional statistics is an indication of topological phases.[7] So far, experiments have confirmed the fractional charge,[8–10] but the direct observation of the fractional statistics remains questionable.[11–16] Recent experiments[11,12] demonstrated the so-called superperiods in the conductance oscillations in a fractional quantum Hall quasiparticle interferometer, which appear to be consistent with fractional statistics.[13,14] However, some theoretical works[15–17] raised subtleties in the interpretations.

A second family of anyons is believed to exist in the fractional quantum Hall state at $\nu=5/2$. The even-denominator state is believed to be a p-wave paired state, known as the Moore-Read state or the Pfaffian state, which supports half-flux quantum vortex excitations.[18] Such particle excitations carry $e/4$ charge each and, when interchanged, not just add a phase factor to the wave function but make the system evolve unitarily in its degenerate (or quasidegenerate for finite systems) ground state manifold. They are dubbed non-Abelian anyons, which are also speculated to exist at $\nu=12/5$. The existence of the non-Abelian anyons, although not confirmed by experiments yet, is of vital importance to topological quantum computing.[19–22]

In theory, the wave functions of quasihole excitations can be written explicitly in analytic functions for both the Laughlin case and the Moore-Read case. They are also exact eigenstates of some special Hamiltonians with short-range two-body and three-body interactions, respectively. Exact diagonalization of finite systems has fruitfully revealed some of these quasihole and/or quasiparticle states.[23] In systems with Coulomb interaction, such ground state descriptions appear to be sufficient even for electrons on a Corbino disk geometry in Abelian cases,[24] as well as for electrons on a disk geometry in the non-Abelian case at $\nu=5/2$.[25] In the latter case, the change of the edge spectrum in the presence of an odd number of non-Abelian anyons at the origin implies the non-Abelian statistics of such excitations. In addition, up to four non-Abelian quasiholes have been induced and oriented tetrahedrally on a sphere, which results in two nearly degenerate states with very similar charge density profile (presumably a topologically protected qubit).[26]

To achieve fault-tolerant quantum computing in the topological fashion, one needs to be able to create individual, paired, or a small cluster of anyons. One of the simplest experimental approaches is probably to use a biased atomic force microscopy (AFM) tip to create and to trap anyons. One may then easily move the anyons localized at the tip to realize braiding to fulfil computation. However, the feasibility of creating anyons at an AFM tip has not yet been systematically studied even on the numerical level. In an earlier work by one of the authors and collaborators,[25] it is demonstrated that a short-range repulsive local potential (as produced by a sharp AFM tip) can induce both $+e/4$ and $+e/2$ quasiholes, depending on the potential strength, in a $\nu=5/2$ system. However, a mixture of long-range Coulomb interaction and short-range three-body interaction is used, and it is not clear whether negatively charged quasiparticles can be created in a similar fashion.

In this work, we study the excitation and trapping of both quasiholes and quasiparticles with a local potential in a microscopic model of fractional quantum Hall droplets with both long-range Coulomb interaction and realistic edge confining potential. We focus on the Laughlin primary filling fraction $\nu=1/3$, although the approach can be applied to other filling fractions, including the intriguing $\nu=5/2$ case,[25,27] to obtain similar results. We find that both positively charged quasiholes and negatively charged quasiparticles can be excited generically by a finite-range tip potential with appropriate sign and strength. We confirm that the edge spectrum of the system is not affected by the presence of a single quasihole, characteristic of its Abelian nature. Our results suggest that it is quite possible to trap individual anyons, as needed in topological quantum computer proposals. We also discuss the stability of anyons when the strength of the confining potential varies.

The rest of the paper is organized as follows. In Sec. II, we consider the short-range hard-core potential, which generates the Laughlin state and the single-quasihole state as exact zero-energy ground states. We consider long-range Coulomb interaction in Sec. III, where we apply tip potentials of δ-function, Gaussian, and exponential forms. We summarize our results and discuss the relevance to experiments in the context of topological quantum computing in Sec. IV.

II. HARD-CORE INTERACTION

In this section, we study the two-dimensional electron system on a disk at filling fraction $\nu=1/3$ with short-range hard-core interaction between electrons in the lowest Landau level (0LL). In Haldane's pseudopotential language, $V_m=\delta_{1,m}$. The Laughlin state[2] at the primary filling factor $\nu=1/3$,

$$\Psi_{1/3}(z_1\cdots z_N)=\prod_{i>j}^{N}(z_i-z_j)^3\exp\left\{-\frac{1}{4}\sum_{i=1}^{N}|z_i|^2\right\}, \tag{1}$$

is the exact ground state with zero energy in the subspace with total angular momentum, $M_{tot}=M_L=3N(N-1)/2$, for N electrons in at least $N_{orb}=3N-2$ orbitals. In fact, it is the zero-energy ground state with the smallest allowed angular momentum; other zero-energy states (for larger N_{orb}) are known as edge states. We plot the density profile of the Laughlin state for ten electrons in 28 orbitals using palette-mapped three-dimensional plot in Fig. 1(a) and, for comparison, along the radial direction in Fig. 1(b).

As Laughlin pointed out, the state with a single quasihole at ξ can be written as

$$\Psi_{1/3}^{qh}(\xi;z_1\cdots z_N)=\prod_{i=1}^{N}(z_i-\xi)\Psi_{1/3}(z_1\cdots z_N). \tag{2}$$

In the disk geometry, ξ can be placed at the origin to preserve rotational symmetry. Obviously, this is a zero-energy ground state in the $M_{1qh}=3N(N-1)/2+N$ momentum subspace for $N_{orb}>3N-2$. In general, there can be additional zero-energy states in the same momentum subspace, with the wave functions being the Laughlin state multiplied by symmetric polynomials of order N. Such degeneracy can be lifted either by limiting $N_{orb}=3N-1$ or by the addition of an impurity potential $H_W=Wc_0^{+}c_0$ at the $m=0$ orbital. In Figs. 1(c) and 1(d), we plot the density profile of the quasihole wave function. A density deficiency around the origin is clearly visible, indicating the presence of a quasihole roughly the size of one magnetic length l_B.

On the other hand, the quasiparticle state of the corresponding Laughlin state is of some ambiguity. There is no zero-energy state obtained in the exact diagonalization at $M_{tot}=M_{1qp}=3N(N-1)/2-N$ because the excitation gap of the Laughlin liquid is finite. Here, it is hard to compare it with the variational quasiparticle wave function,

$$\Psi_{1/3}^{qp}(\xi,z_1\cdots z_N)=\prod_{i=1}^{N}\left[e^{-|z_i|^2/4}\left(2\frac{\partial}{\partial z_i}-\xi^*\right)e^{|z_i|^2/4}\right]\Psi_{1/3}(z_1\cdots z_N), \tag{3}$$

proposed by Laughlin, which is not known as the exact solution of any simple Hamiltonian. We assume that, like the quasihole state, the quasiparticle state is the ground state of the Hamiltonian of interest at the appropriate angular momentum M_{1qp}. Here, we plot the density profile of the ground state as the candidate for the quasiparticle state in Figs. 1(e) and 1(f).

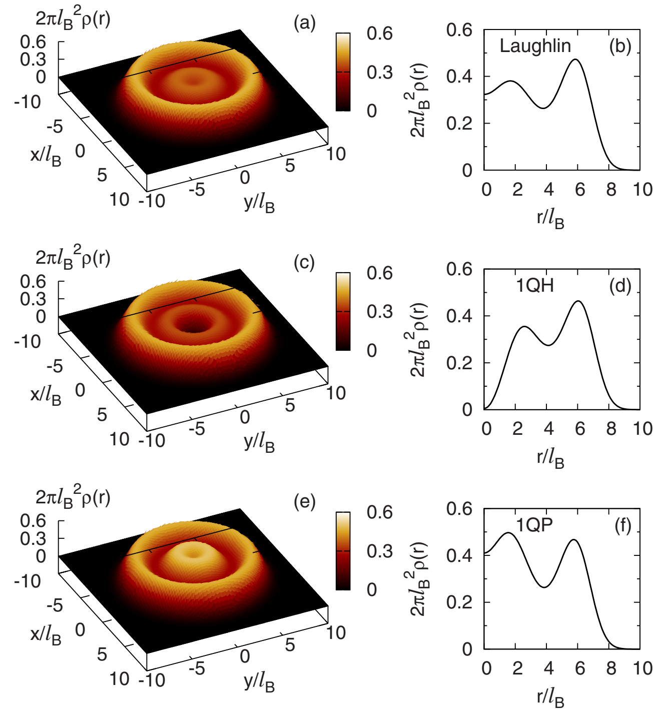

FIG. 1. (Color online) Electron density profiles of the ground states of electrons in the 0LL with hard-core interaction, which generates the Laughlin wave function [Eq. (1)] and the Laughlin quasihole wave function [Eq. (2)] as eigenstates. (a) and (b) show the density profile of the Laughlin state. (c) and (d) show the density profile of the quasihole state (1QH), which is the ground state in the presence of an external potential $W_0c_0^{+}c_0$ at $W_0=0.1$. (e) and (f) show the density profile of the candidate (see text for detail) for the quasiparticle state (1QP) as in Eq. (3).

We plot the accumulated difference of the electron occupation numbers $\Sigma_{i=0}^{m}\Delta n(i)=\Sigma_{i=0}^{m}[n^{qh,qp}(i)-n^{L}(i)]$ between quasihole and/or quasiparticle state (with electron occupation number n^{qh} or n^{qp}) and the Laughlin state (with electron occupation number n^L) in Fig. 2. The dotted line in this figure is the average value, i.e., $1/3$ (or $-1/3$), for the quasiparticle (or quasihole) state. This confirms that there are $\pm e/3$ charged excitations in a Laughlin liquid of $\nu=1/3$. In

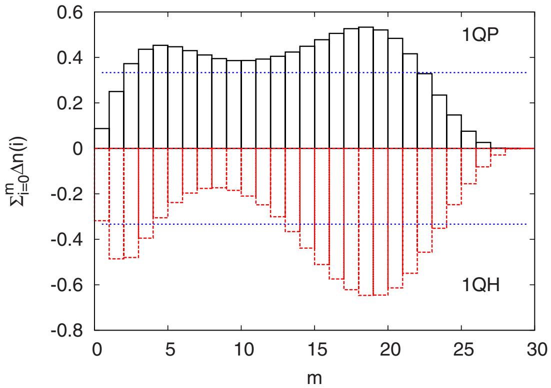

FIG. 2. (Color online) Accumulated difference of the electron occupation number $\Sigma_{i=0}^{m}\Delta n(i)$ between the quasiparticle state (1QP) or the quasihole state (1QH) and the Laughlin state shown in Fig. 1. The difference oscillates around 1/3 (−1/3) for the quasiparticle (quasihole) state, indicating the emergence of a charge $-e/3$ quasiparticle ($+e/3$ quasihole).

the case of the hard-core potential, the size of the $-e/3$ charged quasiparticle ($\sim 3l_B$) is larger than that of the $+e/3$ charged quasihole ($\sim l_B$).

A similar density-profile plot of the Laughlin quasihole and quasiparticle states in spherical geometry has been reported in an earlier numerical work.[23] Studies on the Laughlin quasiparticle excitations in the disk geometry with long-range Coulomb interaction (in the presence of neutralizing background charge to be realistic) have long been absent.

III. COULOMB INTERACTION

In this section, we study the excitations in a Laughlin condensate of N electrons with Coulomb interaction, confined by uniform neutralizing background charge (on a disk of radius R) at a distance d above the electron layer. We use the disk geometry with the symmetric gauge $\vec{A}=(-\frac{By}{2},\frac{Bx}{2})$; the single-particle wave function ϕ_m in the lowest Landau level is

$$\phi_m(z) = (2\pi 2^m m!)^{-1/2} z^m e^{-|z|^2/4}, \tag{4}$$

where $z=x+iy$ is the complex coordinate in the electron layer. Projected to the lowest Landau level, the Hamiltonian in the second quantization language reads

$$H_C = \frac{1}{2}\sum_{mnl} V_{mn}^{l} c_{m+l}^{+} c_{n}^{+} c_{n+l} c_m + \sum_m U_m c_m^{+} c_m, \tag{5}$$

where c_m^+ (c_m) creates (annihilates) an electron at the mth orbital. Here, V_{mn}^{l} are Coulomb matrix elements,

$$V_{mn}^{l} = \int d^2 r_1 \int d^2 r_2 \phi_{m+l}^{*}(\vec{r}_1)\phi_n^{*}(\vec{r}_2)\frac{e^2}{\varepsilon r_{12}}\phi_{n+l}(\vec{r}_2)\phi_m(\vec{r}_1), \tag{6}$$

and U_m the background confining potential,

$$U_m = \frac{Ne^2}{\pi R^2 \varepsilon}\int d^2 r \int_{\rho<R} d^2\rho \frac{|\phi_m(\vec{r})|^2}{\sqrt{|\vec{r}-\vec{\rho}|^2 + d^2}}. \tag{7}$$

In order to study the quasiparticle and quasihole excitations, we include an external local potential H_W, for example, created by an AFM tip. So the complete Hamiltonian is

$$H = H_C + H_W. \tag{8}$$

In the following, we will consider three different forms of H_W: (i) a short-range potential at the origin of the disk $H_W=W_0 c_0^+ c_0$; (ii) a Gaussian potential $H_W=W_g\Sigma_m \times\exp(-m^2/2s^2)c_m^+ c_m$; and (iii) an exponential potential $H_W = W_e\Sigma_m \exp(-m/\xi)c_m^+ c_m$. Throughout the paper, the unit of energy is $e^2/\epsilon l_B$. We would like to point out that the energy of the ground state depends on the strength and the form of the potential, but the qualitative results remain the same.

A. Short-range potential at origin

A short-range potential can be produced by a very sharp AFM tip. By sharp, we mean that the range of the tip potential on the two-dimensional electron gas is smaller than one magnetic length, the size of a single-particle wave function in the lowest Landau level. In this case, we can model the potential by $H_W=W_0 c_0^+ c_0$, located at the origin in our disk geometry. A previous study[25] has applied the short-range potential to create a single $+e/4$ quasihole and two $+e/4$ quasiholes (or $a+e/2$ quasihole) in a model of the fractional quantum Hall liquid at $\nu=5/2$ with Coulomb interaction and an edge confining potential.

To begin with, we apply the same short-range potential H_W to the electron liquid at $\nu=1/3$. We present the results of a system of $N=8$ electrons in 26 orbitals (large enough so that edge excitations have low enough energies). The background charge is still confined to a disk of $R=\sqrt{2N/\nu}=\sqrt{48}$, corresponding to the lowest 24 orbitals, at a distance $d=0.5l_B$ above the electron layer. We expect that the ground state of the Laughlin condensate has a total angular momentum $M_L=3N(N-1)/2=84$, which is found to be the case for zero and small W_0. When we increase W_0 above 0.26 ± 0.01, the total angular momentum of the global ground state jumps from 84 to 92, indicating the excitation of $a+e/3$ quasihole. The density profile of the quasihole state is similar to that found for the hard-core potential [Figs. 1(c) and 1(d)], in which the electron density approaches zero at the origin.

In Fig. 3, we compare the low-energy excitations of the system with and without the quasihole excitation. We identify the edge excitations, labeled by black solid bars, following the approach developed by one of the authors and his collaborators.[28] We observe that in energy relative to the ground state, the edge spectrum looks almost identical with and without the quasihole, implying the Abelian nature of the quasihole. This contrasts to the case of the Moore-Read state, where the presence of $a+e/4$ quasihole changes the fermionic edge excitations. The numbers of the edge states (including the ground state) are 1, 1, 2, 3, and 5 for $\Delta M=0-4$, as expected by the chiral boson edge theory.[29,30]

The short-range potential is useful to generate a single $+e/3$ quasihole if the edge confinement is not too strong. However, since it only affects the local potential at a single orbital, a second quasihole cannot be induced, since one cannot deplete more than (on average) 1/3 charge in a single

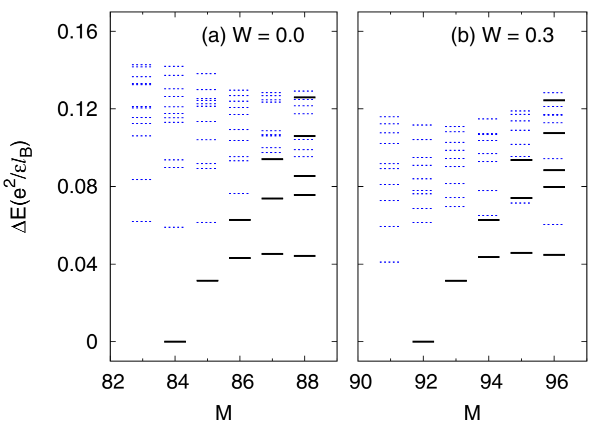

FIG. 3. (Color online) Low-lying energy spectra obtained by exact diagonalization for eight electrons interacting via Coulomb repulsion in 26 orbitals in the 0LL (a) with and (b) without a quasihole at the origin. Edge states are marked by black solid bars, while bulk states by blue dotted bars. The quasihole at the origin is excited by a short-range potential $W_0 c_0^+ c_0$ with $W_0=0.3$. The similarity of the two edge spectra confirms the Abelian nature of the quasihole.

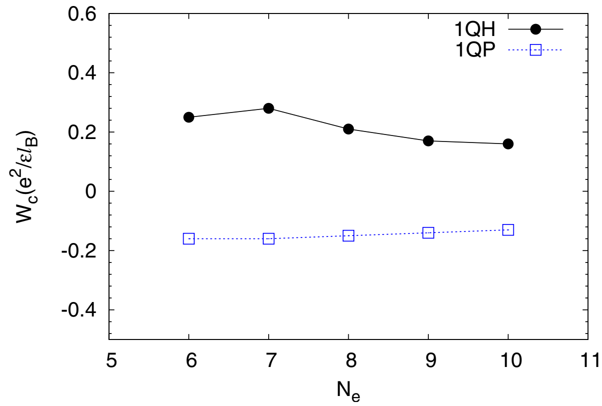

FIG. 4. (Color online) Threshold strength W_c for the excitation of a single quasihole (black solid dots) and a single quasiparticle (blue empty squares) in $\nu=1/3$ systems with six to ten electrons with Coulomb interaction. The quasihole or quasiparticle is trapped by a Gaussian potential $H_W=W_g\Sigma_m \exp(-m^2/2s^2)c_m^+ c_m$ with width $s=2$.

orbital in this case. For the same reason, the short-range potential does not support a single quasiparticle (charged $-e/3$), as one can see from Fig. 2 that a quasiparticle occupies several orbitals, unlike a quasihole. Therefore, we proceed to study local potentials with a longer range.

B. Gaussian-shaped potential

We now considered the Gaussian-shaped potential $H_W=W_g\Sigma_m \exp(-m^2/2s^2)c_m^+ c_m$, i.e., the potential has a value $W_g \exp(-m^2/2s^2)$ on the mth orbital. The width of the potential is s, while the strength of the potential W_g. In the limit of $s\rightarrow 0$, the Gaussian potential evolves into the short-range δ potential discussed in the previous subsection.

For fixed $s=2$ and $d=0.5l_B$, we vary W_g to study the change of the total angular momentum of the global ground state. For example, in a system of $N=10$ electrons in 30 orbitals, we find that the total angular momentum M_{tot} jumps from $M_L=3N(N-1)/2=135$ to $M_{1qh}=3N(N-1)/2+N=145$ at $W_g=0.16\pm 0.01$, indicating the presence of one quasihole at the origin. For an attractive potential, M_{tot} drops from $M_L=135$ to $M_{1qp}=3N(N-1)/2-N=125$ at $W_g=-0.13\pm 0.01$, indicating the emergence of a quasiparticle at the origin. Figure 4 shows the value of W_g at which M_{tot} changes for systems with six to ten electrons. We find that the threshold values for the generation of one quasihole or one quasiparticle approach a constant value of roughly ± 0.15 as the system size increases. It is not surprising that the threshold value is of the same energy scale as the bulk energy gap, but it also depends on the detail of the potential.

Figure 5 shows the electron density profiles for (a) the Laughlin condensate, (b) the one-quasihole state for $W_g=0.2$, and (c) the one-quasiparticle state for $W_g=-0.2$. The density accumulation or depletion at the origin indicates the presence of a quasiparticle or quasihole. Compared to the case of the hard-core potential, the quasihole is slightly larger, while the quasiparticle is smaller with a well-defined peak at the origin. Therefore, in the more realistic case with Coulomb interaction (and a not too narrow tip), the quasihole state and the quasiparticle state have roughly the same perturbation to the Laughlin condensate except for the opposite signs, suggesting a quasiparticle-quasihole symmetry.

Since the quasiparticle state we obtain for the Gaussian potential in systems with Coulomb interaction cannot be easily compared with the variational wave function [Eq. (3)], we want to make sure that it is not a stripe phase, which arises commonly in systems with Coulomb interaction. In Fig. 5(c), we find that the electron density of the quasiparticle state has a large value around the origin and edge. A stripe phase of $N=10$ electrons in 30 orbitals with a somewhat similar density distribution and the same total angular momentum can be represented by a binary string $|\Psi_{SP}\rangle = |110000000000111111110000000000\rangle$ (a Slator determinant), in which each digit specifies the corresponding single-electron orbitals (from 0 to 29) being occupied (1) or not (0). Therefore, we wish to answer the question how close the ground state with $M_{tot}=125$ is to the stripe phase. For this, we plot the lowest four excitation energies (energy difference between the lowest four excited states and the ground state in the subspace of $M=125$) as a function of W_g in Fig. 6(a). Obviously, there is no crossing and/or anticrossing between the ground state (which we identify as the quasiparticle state) and the first excited states as $|W_g|$ increases. This is very different from the behavior of the next three excited states, which can get very close in energy. We further calculated the overlaps between the lowest two energy states and the stripe state as a function of W_g in Fig. 6(b). While the overlap is increasing for the ground state, it is only about 5% for $W_g \sim W_g^c=-0.13$ when the ground state in the $M=125$ subspace becomes the global ground state. We therefore conclude that the ground state is unlikely the stripe state.

We extend our calculation to a grid on the area defined by $-2\leqslant W_g \leqslant 2$ and $0<s<3.5$ for the Gaussian potential $W_g\Sigma_m \exp(-m^2/2s^2)c_m^+ c_m$. We choose the strength of edge confinement by placing the background charge at $d=0.5l_B$

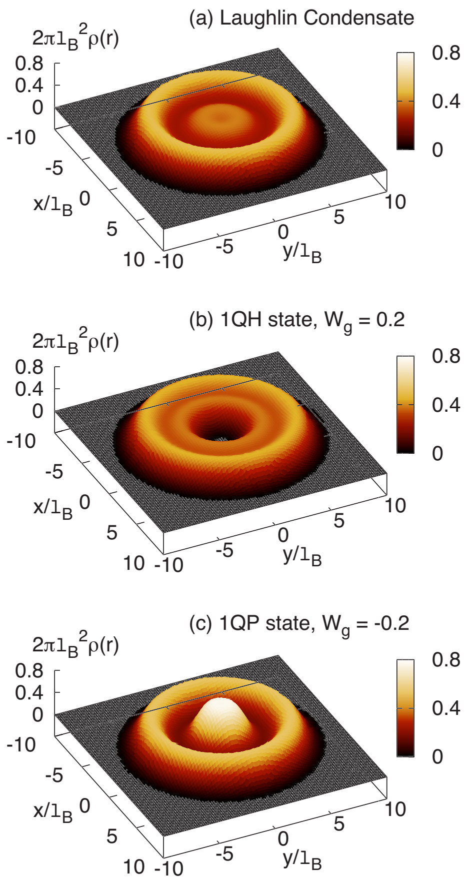

FIG. 5. (Color online) Electron density profiles of the ground states of electrons in the 0LL with Coulomb interaction, obtained by numerical diagonalization. We consider a system of ten electrons in 30 orbitals and apply a Gaussian potential $H_W = W_g \Sigma_m \times \exp(-m^2/2s^2) c_m^+ c_m$ with $s=2$ to trap a quasihole or a quasiparticle. These ground states are (a) the Laughlin condensate at $W_g = 0$, (b) the one-quasihole state at $W_g = 0.2$, and (c) the one-quasiparticle state at $W_g = -0.2$.

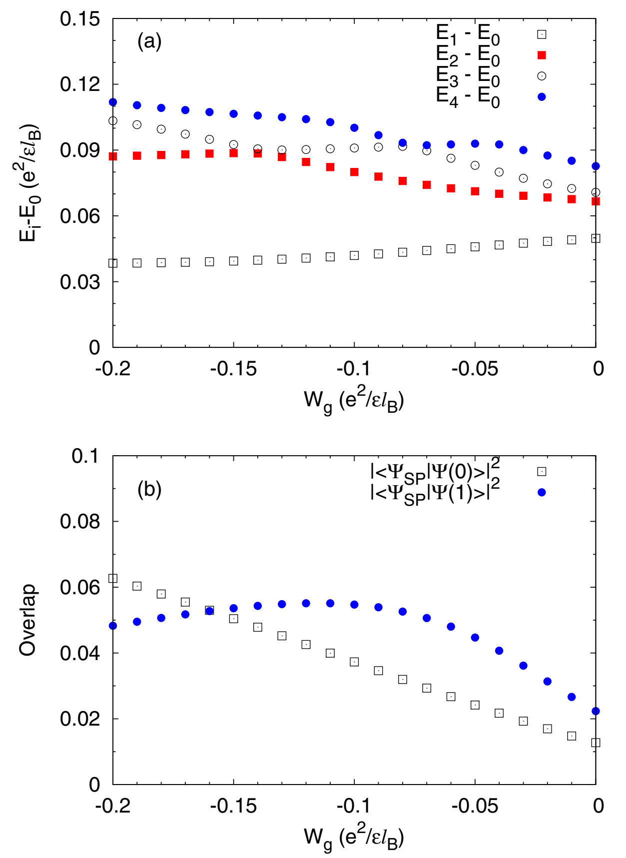

FIG. 6. (Color online) Evolution of (a) the excitation energies of the lowest four excited states and (b) the overlaps of the lowest two states with the stripe phase $|\Psi_{SP}\rangle = |110000000000111111110000000000\rangle$ at total angular momentum $M = 125$, for a system of ten electrons in 30 orbitals with Coulomb interaction and a Gaussian potential $W_g \Sigma_m \exp(-m^2/2s^2) c_m^+ c_m$ with $s=2$. The neutralizing confining charge is located at a distance $d = 0.5 l_B$ above the electron layer. The lack of ground state energy level crossing or anticrossing and the relatively small overlap with the strip state suggest that the ground state can be interpreted as the one-quasiparticle state, rather than the strip state.

(stronger confinement) and $d = 1 l_B$ (weaker confinement). The results of the ground state for eight electrons are plotted in Fig. 7. Generically, we can divide the parameter space into five regions: the Laughlin condensate $[M_{tot} = 3N(N-1)/2]$, the one-quasihole state $[M_{tot} = 3N(N-1)/2 + N]$, the one-quasiparticle state $[M_{tot} = 3N(N-1)/2 - N]$, beyond one-quasihole state [ground states with $M_{tot} > 3N(N-1)/2 + N$], and beyond one-quasiparticle state [ground states with $M_{tot} < 3N(N-1)/2 - N$]. We have also done the calculation for $N = 10$ with similar results, but on a coarser grid. In particular, we do observe the ground state with $M_{tot} = 3N(N-1)/2 + 2N = 155$ for $N = 10$, consistent with the angular momentum for a two-quasihole state. We do not find any global ground state with $M_{tot} = 3N(N-1)/2 - 2N = 115$ (consistent with that of a two-quasiparticle state), but at 117. One might tempt to speculate this as one of the two quasiparticles moving away from the origin. Nevertheless, in such small systems, it is unnecessary and most likely unreliable to emphasize multiple quasiparticle and quasihole excitations, so we simply mark the regions with $M_{tot} > 3N(N-1)/2 + N$ and $\Delta M_{tot} < 3N(N-1)/2 - N$ by "beyond 1QH" and "beyond 1QP," respectively, and do not proceed further.

The main difference between $d = 0.5 l_B$ and $d = 1.0 l_B$ occurs along the boundaries of quasiholes, not along the quasiparticle boundaries. This, we believe, is due to the fact that for a fixed number of electrons, the quasihole states (not the quasiholes themselves) have larger size than the quasiparticle states, thus more susceptible to the edge confinement. The difference is more evident at smaller s (sharper tips). In particular, a δ tip can excite a quasihole in the case of $d = 1.0 l_B$, but not in the case of $d = 0.5 l_B$ for not too large W_g. This, as illustrated in Fig. 7, suggests that a finite width $s \approx 2 l_B$ may be more robust for the excitation of quasiholes and quasiparticles.

C. Exponential-shaped potential

In this section, we discuss the exponential-shaped potential $H_W = W_e \Sigma_m \exp(-m/\xi) c_m^+ c_m$. In real space, this corresponds to a Gaussian potential $V(z) = W_g^r e^{-|z|^2/2\sigma^2}$, which may not be too difficult to prepare in experiments. Explicitly, by

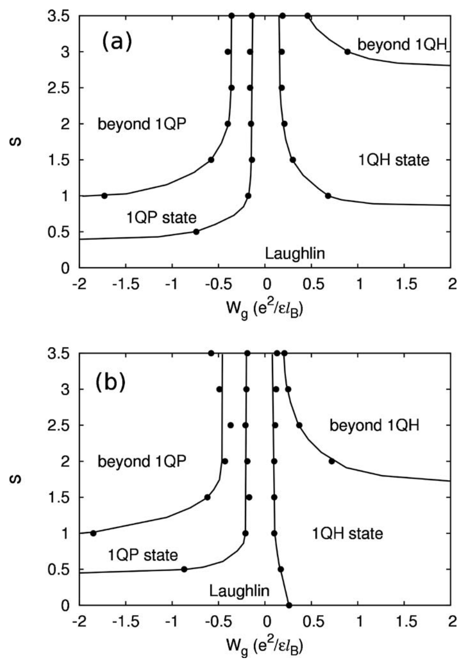

FIG. 7. Ground state phase diagram for the system of eight electrons in 24 orbitals with Coulomb interaction in the presence of the Gaussian tip potential with strength W_g and width s. The confining charge is located at distances (a) $d=0.5l_B$ and (b) $d=1l_B$ above the electron layer. Generically, we can divide the parameter space into five regions: the Laughlin condensate, the one-quasihole (1QH) state, the one-quasiparticle (1QP) state, beyond 1QH state, and beyond 1QP state, according to their corresponding ground state total angular momenta. The difference between (a) and (b) suggests that the energy of the quasihole state is more susceptible to edge confinement.

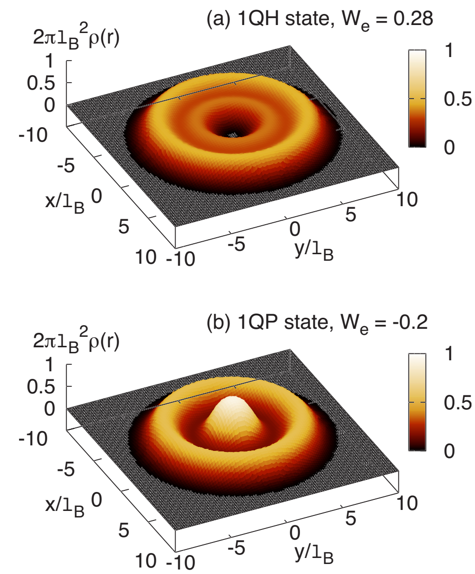

FIG. 8. (Color online) Electron density profiles for (a) the one-quasihole state (W_c=0.28) and (b) the one-quasiparticle state (W_e =−0.2) for the exponential potential $H_W=W_e\Sigma_m \exp(-m/\xi)c_m^+c_m$ with $\xi=1/\ln 2$ (or $\sigma=l_B$ in real space). The system has ten electrons in 30 orbitals with Coulomb interaction. The neutralizing confining charge is located at a distance $d=0.5l_B$ above the electron layer.

projecting the real-space potential into the lowest Landau level, we obtain the matrix elements,

$$\langle \phi_m|V|\phi_m\rangle = \frac{W_g^r}{2\pi 2^m m!}\int_0^\infty \exp - \frac{l_B^2+\sigma^2}{2\sigma^2}\frac{|z|^2}{l_B^2}\frac{|z|^{2m}d^2z}{l_B^{2m+2}}$$
$$= W_g^r\left(\frac{\sigma^2}{l_B^2+\sigma^2}\right)^{m+1} = W_e e^{-m/\xi}, \tag{9}$$

where the decay length is $\xi=1/\ln(1+l_B^2/\sigma^2)$ and the effective strength $W_e=W_g^r\sigma^2/(\sigma^2+l_B^2)$. Again, $|\phi_m\rangle$ is the lowest Landau level wave function with angular momentum m.

After applying the exponential potential $H_W=W_e\Sigma_m \times\exp(-m/\xi)c_m^+c_m$ with $\xi=1/\ln 2$ (or $\sigma=l_B$ in real space), we are also able to trap a single quasihole or a single quasiparticle. Again, we consider a system of ten electrons in 30 orbitals, with neutralizing confining charge located at a distance $d=0.5l_B$ above the electron layer. The electron density profiles for the one-quasihole state and the one-quasiparticle state are plotted in Fig. 8. The density profiles look very similar to those for the Gaussian potential discussed in the previous section (Fig. 5). Like the Gaussian case, the quasiparticle state and the quasihole state have roughly the same density perturbation (but with opposite signs) to the Laughlin condensate.

IV. CONCLUSION AND DISCUSSION

To summarize, we study the trapping of quasiholes and quasiparticles by a local potential (e.g., induced by an AFM tip) in a microscopic model of fractional quantum Hall liquids with short-range hard-core interaction or long-range Coulomb interaction with an edge confining potential due to neutralizing background charge. We find, in particular, that at the Laughlin filling faction $\nu=1/3$, both quasihole and quasiparticle states can be energetically favorable for the ground state of the Coulomb system for tip potentials of various shapes and strengths. The presence of the Abelian quasihole has no effect on the edge spectrum of the quantum liquid, unlike in the non-Abelian case when fermionic excitations are present.

Although quasiholes and quasiparticles can emerge generically in the system, its stability depends on the strength of the confining potential and the strength and the range of the tip potential. Experimentally, the quantum Hall plateau at $\nu=1/3$ was found in a high magnetic field (∼15 T).[1] In this case, the magnetic length $l_B\approx 70$ Å. Based on our microscopic calculation, we estimate an optimal range of the tip potential to be 140 Å. The size falls in the right range of AFM tip size under current technology. The Laughlin condensate in the context of topological quantum computing is of less interest due to its Abelian nature, although it can be used for topological quantum memory. Nevertheless, it is much easier to model in numerical studies than the non-Abelian Moore-Read state,[27] and the even more complicated Read-Rezayi (parafermion) states.[31] We expect that the results found here can help for the excitation and trapping of quasiholes or quasiparticles in the Moore-Read case in future experiments. In the Moore-Read case at filling fraction of

$5/2$, a smaller magnetic field of ~ 5 T is usually applied. Thus, with a longer magnetic length, we can have even wider tips, which should not be a technical challenge. The trapping of anyons might also be realized in optical lattices of atoms[32,33] or polar molecules.[34]

With the well-known difficulties of the exact diagonalization method in highly entangled systems such as the fractional quantum Hall liquids, the search for the ground states with a few parameters is a time-consuming job. The Moore-Read case is even more complicated, since the even-denominator state has a smaller excitation gap and is competing with stripe phases.[27] One might wish to develop more efficient numerical methods to approach the ground state properties. One development in recent years is the application of density-matrix renormalization group method[35–38] to the fractional quantum Hall systems. We implement the method in the disk geometry with results in excellent agreement with exact diagonalization in small systems.[39] However, we find that the time to reach convergence (especially near the origin) in larger systems is impractically long for the extensive search for the ground states discussed in the current paper.

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